equations. Boole's system recognizes both. Unlike other revolutionary logical innovators, Boole's greatness as a logician was recognized almost immediately. In 1865, hardly a decade after Boole's 1854 *Laws of Thought* and not even a year after Boole's tragic death, his logic was the subject of a Harvard University lecture "Boole's Calculus of Logic" by C.S. Peirce, America's most creative native logician. Peirce opened his lecture with these prophetic words:

> Perhaps the most extraordinary view of logic which has ever been developed with success is that of the late Professor Boole. His book ... *Laws of Thought*. ... is destined to mark a great epoch in logic; for it contains a conception which in point of fruitfulness will rival that of Aristotle's *Organon*.
> (Peirce 1982: 223–4)

Peirce was among the earliest logicians to discern Boole's achievement.

Aristotle's system recognized only four logical forms of propositions, each involving exactly two (non-logical) terms; today infinitely many are recognized, with no limit to the number of terms occurring in a single proposition. In fact, as early as his famous 1885 paper "On the Algebra of Logic: A Contribution to the Philosophy of Notation," also reprinted in the 1992 Houser-Kloesel volume, Peirce recognized in print simple propositions having more than two terms (1992: 225–6). Examples are the triadic proposition that the sign "7" denotes the number seven to the person Charles and the tetradic proposition that one is to two as three is to six. Peirce revisits the topic in his 1907 manuscript "Pragmatism," printed in the 1998 Houser-Kloesel volume (1998: 407–8), where he presented his now well-known triadic analysis of propositions about giving as "The person Abe gives the dog Rex to the person Ben."

Aristotle's system recognized only three patterns of immediate one-premise deductions and only four patterns of immediate two-premise deductions; today many more are accepted. In particular, he never discerned the fact pointed out by Peirce that to every pattern of deduction there is a proposition to the effect that its conclusion follows from its premises. Peirce (1992: 201) called them *leading principles*. It never occurred to Aristotle to include in his system such propositions as, for example, that given any two terms if one belongs to all of the other then some of the latter belongs to some of the former.

The simple linear chain structures of Aristotle's deductions have been augmented by complex non-linear structures such as branching trees and nested linear chains. Moreover, his deductive logic has been subjected to severe criticism. Nevertheless, the basic idea of his demonstrative logic, the truth-and-consequence theory of demonstration, which was fully accepted by Boole, has encountered little opposition in its more than 2,000-year history. It continues to enjoy wide acceptance in the contemporary logic community. Perhaps ironically, Peirce never expressed full acceptance and, in at least one place, he seems to say that diagrams are essential not only in geometrical demonstrations (1998: 303) but in all demonstration (1998: 502).

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JOHN CORCORAN

LOGIC: EXPERIMENTAL

"Experimental logic" is a name that John Dewey gave to his theory of inquiry, especially as it was developed in *Essays in Experimental Logic* (1916), *How We Think* (1910), and *Logic: The Theory of Inquiry* (1938). In the preface to his 1916 volume Dewey indicated that his experimental logic contained "psychological phases" and that those phases were written from the standpoint of behaviorism. This statement deserves special attention on two counts.

First, some of Dewey's critics, including Charles S. Peirce, accused Dewey of confusing logical method (how people ought to think) with an analysis of psychological processes of thinking (how people do in fact think). For Dewey, however, logical forms are normative for inquiry in the sense that they have arisen within the context of inquiry into inquiry and have been demonstrated to be capable of producing judgments that are true in the sense of being both warranted and assertible. Just as farm machinery has developed as a result of inquiry into farming practices, the "machinery" of logic is the product of inquiry into the practice of inquiry. The tools of logic, like the tools of farming, are constructed artifacts; they are neither discovered as entities that existed prior to inquiry nor are they invented out of nothing. In each case there is modification, and modification of modification, of naturally occurring existents.

Second, experimental logic must be understood from the standpoint of behaviorism. Inquiry is public, organic behavior. Moreover, because inquiry involves signs, abstract entities, and other cultural artifacts, it is social behavior. Inquiry is also social behavior in the moral sense that it allows individuals to "rehearse" courses of action before committing themselves to decisions that might be harmful to themselves and others. This type of behaviorism was thus quite different from that of John B. Watson, which depended on what Dewey regarded as a discredited account of stimulus–response (S–R) theory, an unwarranted reduction of psychology to physiology, and treatment of psychology as if it were a science of the behavior of individuals.

Dewey argued that there is continuity within inquiry. Methods utilized by control of inquiry in the sciences and control of inquiry in commonsense affairs are basically the same: they differ only in degree of complexity and type of subject matter. Both types of inquiry are concerned with adjustment to situations that are social in nature.

Experimental logic is thus more comprehensive than symbolic or mathematical logics, which have tended to separate formal systems of proof from the broader concerns of scientific method. Dewey considered formal logic an essential part of the broader processes of inquiry, but rejected attempts to identify inquiry with formal logic *simpliciter*.

Dewey noted that experimentation is neither simply a practical convenience nor just a means of modifying states of mind. Experiment is required to organize and deploy the data that are employed to warrant inferences, since experiences per se are not sufficient to this task. Experiment is also required to eliminate irrelevant existential material and to seek additional material that may be relevant to the problem at hand.

More specifically, Dewey thought that the process of inquiry – quotidian as well as scientific – involves a more or less well-defined sequence. Once an indeterminate situation has become determinate in the sense that it is recognized as in need of reconstruction in a manner that settles doubt, four types of relations enter into the process. In the first there is recognition of involvement of existential conditions with other existential conditions. In the second, some existent is treated as a sign or symbol of another existent, giving rise to inference. In the third, there is a relationship of implication between symbols in their role as symbols. It is at this level of *abstracta* relating to other *abstracta* that symbolic logic and scientific hypotheses operate and are developed. Finally there is the relation of symbols (hypotheses and other *abstracta*) to existential affairs that Dewey terms "reference." This is the stage at which hypotheses are tested against existential conditions that initiated the problem under consideration.

The process just described is Pragmatic in the sense that it involves inquiry into the practical consequences that the inquirer conceives the object of his or her conceptions to have; it is instrumental in the sense that logical entities, hypotheses, and other *abstracta* are treated as tools or instruments of inquiry (and not as having independent ontological status as essences), and it is experimental in the sense that it involves "the art of conducting a sequence of observations in which natural conditions are intentionally altered and controlled in ways which will disclose, discover, natural subject-matters which would not otherwise have been noted."

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LARRY A. HICKMAN

LOGIC: INDUCTIVE

Logic is the study of the quality of arguments. An argument consists of a set of premises and a conclusion. The quality of an argument depends on at least two factors: the truth of the premises, and the strength with which the premises confirm the conclusion. The truth of the premises is a contingent factor that depends on the state of the world. The strength with which the premises confirm the conclusion is supposed to be independent of the state of the world. Logic is only concerned with this second, logical factor of the quality of arguments.

Deductive logic classifies arguments into two kinds: those where the truth of the premises guarantees the truth of the conclusion, and those where they do not. The former are called deductively valid, and the premises are said to logically imply the conclusion. The latter arguments are called deductively invalid. So the deductive-logical explication of the logical factor of the quality of an argument is the qualitative yes-or-no concept of deductive validity.

Inductive logic aims at a more lenient explication of the logical factor of the quality of an argument. It comprises deductive validity as a special case. The reason is that the conclusions we are normally interested in are too informative to be logically implied by premises we can know. For instance, no set of premises about the past and present logically implies a conclusion about the future. Inductive logic usually aims at a quantitative explication of the logical factor of the quality of an argument, viz. the degree to which the premises confirm the conclusion.

Hempel (1945) made one of the earliest attempts to develop a formal logic of qualitative confirmation. His goal of constructing a purely syntactical definition of confirmation is shared by Carnap (1962), who goes beyond Hempel by aiming at a quantitative concept of degree of confirmation. Carnap bases his inductive logic on the theory of probability (Kolmogorov 1956). Due to Goodman's (1983) "new riddle of induction" there is consensus nowadays that a purely syntactical definition of (degree of) confirmation cannot be adequate. However, the use of probability theory has been a central feature of inductive logic ever since.

A "probability measure" is a real-valued function on a language or field of propositions that is (1) non-negative, (2) normalized, and (3) additive. So every proposition receives a non-negative probability; the tautological proposition receives probability 1; and the probability of the union or disjunction of two disjoint or incompatible propositions is the sum of the probabilities of the two propositions. The conditional probability of one proposition given another proposition is defined as the ratio of the probability of the intersection or conjunction of the two propositions divided by the probability of the second proposition. Obviously this makes sense only if the second proposition receives positive probability.

In inductive logic conditional probability is usually put to use in the following way (Carnap 1962; Hawthorne 2005; Skyrms 2000). The "degree of absolute confirmation" of a conclusion by a set of premises relative to a probability measure on a field of propositions is defined as the conditional probability of the conclusion given the (conjunction of the) premises. For more see Huber (2005).

It is important to note that this definition renders degree of confirmation relative to a probability measure on a language or field of propositions that include the premises and the conclusion. The difference between the Carnapian approach (Carnap 1962) and more modern approaches (Hawthorne 2005; Skyrms 2000) now can be put as follows. Carnap sought to come up with one single logical probability measure, whereas modern writers consider (almost) any probability measure as admissible from a purely logical point of view.

The notion of deductive validity is a three-place relation between a set of premises, a conclusion, and a language that includes the premises and the conclusion. By trying to define a unique logical probability measure for each language, Carnap in effect tried to define degree of confirmation in a similar fashion as a three-place relation between a set of premises, a conclusion, and a language. Modern theories of confirmation differ in this respect, because they construe confirmation as a four-place relation, thus making explicit the probability measure. Fitelson (2005) still considers this to be a logical relation.

Carnap (1962) also proposed a definition of qualitative confirmation, where the idea is that premises confirm a conclusion if they raise the probability of the conclusion. A conclusion is incrementally confirmed by a set of premises relative to a probability measure on a field of propositions if and only if the conditional probability of the conclusion given the premises is higher than the unconditional probability of the conclusion.

As indicated by the qualifiers "absolute" and "incremental," we have here two different concepts of confirmation. The quantitative concept of absolute confirmation is explicated by the conditional probability of the conclusion given the premises. Absolute confirmation thus consists in high conditional probability, and the qualitative concept of absolute confirmation is to be defined as follows. A conclusion is absolutely confirmed by a set of premises relative to a probability measure on a field of propositions if and only if its degree of absolute

confirmation is sufficiently high. Incremental confirmation, on the other hand, focuses on increase in probability. Therefore the quantitative concept of incremental confirmation is to be defined as the degree to which the premises increase the probability of the conclusion, i.e. the difference between the unconditional probability of the conclusion and the conditional probability of the conclusion given the premises.

As noted by Fitelson (1999), there are many non-equivalent ways to measure degree of incremental confirmation. Earman (1992) discusses the distance measure, which subtracts the unconditional probability of the conclusion from its conditional probability given the premises. Joyce (1999) and Christensen (1999) propose a measure which subtracts the conditional probability of the conclusion given the negation of the premises from its conditional probability given the premises.

In a different context, Carnap and Bar-Hillel (1952) propose to measure the informativeness of a conclusion by the probability of its negation. Hempel and Oppenheim (1948) suggest measuring the extent to which the conclusion informs us about the premises by the conditional probability of the negation of the conclusion given the negation of the premises. This is relevant since it turns out that the above-mentioned measures of incremental confirmation are aggregates of the degree of absolute confirmation and the informativeness in the respective senses. More precisely, incremental confirmation is proportional to expected informativeness. Different measures of incremental confirmation differ in the way they measure informativeness.

We have thus detected a third factor of the quality of an argument: the informativeness of the conclusion. This is not surprising. After all, the informativeness of the conclusion was the very reason why we were considering more lenient standards than deductive validity in the first place. Note also that the informativeness of the conclusion is as much a logical factor as is the degree to which the premises confirm the conclusion. For both factors are determined once the premises, the conclusion, and the probability measure on the field of propositions are specified. In fact, this opens the door to render all factors of the quality of an argument to be logical; for we can now also consider the probability that the premises are true.

So far we have been engaged in conceptual analysis, where we appeal to intuitions as the data against which to test various proposals for a definition of confirmation. The assumption is, of course, that the concept we are explicating is important. Surely it is a good thing for a hypothesis to be confirmed by the available data. Surely we should strive to list premises that confirm the conclusion we are arguing for. Inductive logic is important, because it is a normative theory. Yet conceptual analysis does not provide the resources to justify a normative theory. Appeals to intuitions do not show why we should prefer "well-confirmed" hypotheses to other hypotheses, and why we should provide inductively strong rather than any other arguments.

The analogy to deductive logic again proves helpful. The rules of deductive logic are norms that tell us how we should argue deductively. As any other set of norms, it needs to be justified. Contrary to Goodman (1983), the rules of deductive logic are not justified, because they adequately describe our deductive practices. They do not. The rules of deductive logic are justified *relative to* the goal of arguing truth preservingly, i.e. in such a way that the truth of the premises guarantees the truth of the conclusion. The results that provide the justification are known as soundness and completeness. Soundness says that every argument we obtain from the rules of deductive logic is such that truth is preserved when we go from the premises to the conclusion. Completeness states the converse. Every argument that has this property of truth preservation can be obtained from the rules of deductive logic. So the rules of deductive logic are justified relative to the goal of truth preservation. The reason is that they further this goal insofar as all and only deductively valid arguments are truth-preserving.

What is the goal-inductive logic supposed to further – relative to which it can be justified? Surely it includes truth. However, as Hume (1739) argues, it is impossible to justify induction relative to the goal of truth. His argument assumes that justifying induction means providing a deductively valid or an inductively strong argument with knowable premises for the conclusion that induction will always lead to true conclusions. As noted by Reichenbach (1938), there are deductively valid arguments for other conclusions that may show that induction furthers the goal of truth to the extent this is possible. Similar results obtain for absolute confirmation, where it can be shown that the conditional probability of a conclusion given the premises converges to its truth value when more and more premises are learned.

However, if obtaining true conclusions were the only goal induction is supposed to further, induction could be replaced by deduction. All that is logically implied by what we know is guaranteed to be true. We do not need to go beyond the premises to satisfy the goal of truth. The reason we nevertheless do go beyond what is logically implied by the premises is that we aim at more than mere truth: we aim at informative truth. It is this very feature that makes us strive for a more lenient explication of the logical factor of the quality of arguments in the first place; and without it Hume's problem of the justification of induction would not even get off the ground. Thus, the important question is whether and in which sense inductive logic can be justified relative to the goal of informative truth. One answer is given by Huber (2005). There it is shown that incremental confirmation in the sense of the above-mentioned measures converges to the most informative among all true conclusions when more and more premises are learned.

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FRANZ HUBER

LOGIC: INFORMAL

Informal logic is, as the name of the subject suggests, not formal logic. Unlike formal logic, it does not consist of precise techniques for determining whether an argument is valid, i.e. whether the truth of the premises of an argument necessitates the truth of the conclusion of that argument. Informal logic, taught primarily in critical thinking courses, consists of a number of techniques – other than those techniques studied in formal logic – for identifying arguments as either flawed or successful arguments.

The most well-known technique in informal logic, historically, has been the use of descriptions of common fallacies. A fallacy is a form argument that generally contains premises that do not give adequate support to the conclusion of the argument. Informal logic allows one to use descriptions of common fallacies to identify arguments as flawed. Among the most common and well-known fallacies are ad hominem fallacies, and ad populum fallacies.

Ad hominem fallacies contain premises that are not directed toward the issue in question, but rather directed toward the person who raises the issue. A critic of the war in Iraq, for example, who supports her anti-war position on the basis of the claim that George W. Bush is an unintelligent person, is committing an ad hominem fallacy.

The premises of ad populum fallacies concern the popularity of the conclusion to argue for that conclusion. If one argues that it is permissible for corporations to use sweatshop labor on the basis of the claim that the majority of US citizens are unconcerned regarding the conditions in factories making inexpensive clothes, then one has appealed to ad populum to make this point.

It would be a mistake to think that informal logic consists solely of the technique of identifying fallacies. Courses in informal logic also typically cover issues related to clarity in writing and speaking, teaching students how to avoid vagueness, ambiguity, and obscurity. A central concern of informal logic is the structure of arguments. Techniques such as diagramming of arguments are employed for the purpose of identifying which statements in a discourse are premises, which statements are conclusions, and whether the premises support their conclusions separately or in conjunction with each other. A related technique is identifying enthymemes, arguments with unstated premises or conclusions. An example of an enthymeme is the following: Iran poses a threat to the security of the United States, therefore the United States ought to invade Iran. The unstated premise is that if a country poses a threat to the security of the United States, the United States ought to invade that country.

Informal logic also involves techniques for identifying the premises of arguments as acceptable premises. Whereas formal logic only studies the relationship between premises and conclusions, informal logic studies whether a given premise ought to be accepted or rejected in light of the available evidence. To cite a fairly simple example, premises from an unreliable source, such as a celebrity tabloid magazine, ought not to be given as much credence as a premise that is obtained from a (generally) reliable source, such as the *Washington Post*.

Through the study of these topics, informal logic contributes to the study of logic generally, the study that distinguishes good and bad arguments.

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FRITZ J. MCDONALD

LOGIC: MATHEMATICAL

Mathematical logic is a field of mathematics that includes the study of symbolic logic and its metatheory. The latter includes proof theory (the study of deducibility relations among sentences in a formal language, where deducibility is defined syntactically) and model theory (the study of interpretations of formal languages, where interpretations are specified in extensional settheoretic terms suited for the study of the language of mathematics). Key metatheoretic questions regarding a given formal language concern correspondences between syntactic deducibility relations among sentences in that language and semantic consequence relations among sentences as determined by interpretations of the language. A proof system for the given language is *complete* with respect to a given class of interpretations just in case any semantic consequence of a set of premises in those interpretations is syntactically provable from those premises. Conversely, the proof system is *sound* with respect to a given class of interpretations just in case any sentence that is syntactically provable from a given set of premises is respectively a semantic consequence of those premises. Besides issues of soundness and completeness, mathematical logic is also concerned with the expressive capabilities of formal languages (what concepts they can and cannot explicitly distinguish; what properties and relations they can and cannot categorically define).

Computability theory falls within the scope of mathematical logic insofar as certain classes of algebraic grammars are associated with respective computational capabilities. Inductive logic, if there are such logics, involves the use of statistics and probability theory; but while statistics and probability theory employ sophisticated mathematical machinery, this does not place inductive inference within the scope of mathematical logic as such. Mathematical logic is restricted rather to a study of formal systems of deductive inference and algorithmic computation, particularly as such formal systems provide potential mathematical models of the expressive and *deductive* capabilities of natural language.

While sometimes characterized as a science of correct reasoning, formal logic is in fact at best a study of correct deductive reasoning. A standard development of formal deductive logic begins with propositional logic and proceeds to introduce various refinements. The primitive elements in *propositional logic* are sentential variables along with an expressively complete set of truth-functional connectives. The main concern here is with arguments (provability and/or consequence relations) whose validity depends on truth-functional operations such as negation, conjunction, disjunction, material implication, and the like. Semantically, one assumes that the world consists of facts and that sentences will be true or false relative to a given domain, insofar as they do or do not express facts in that domain. Various modifications are possible: e.g. truth-value gaps or multiple truthvalues are possible; or one may permit infinitely long well-formed sentences.

One the other hand, firstorder predicate (quantificational) logic allows that sentences will have internal structures reflecting more detailed ontological commitments. The assumption here is that the world consists of objects having various properties and standing in various relations. Facts are at bottom determined by which objects have what properties or stand in which relations. The language of firstorder logic in this case requires predicate (relational) symbols, names, individual variables, perhaps functional symbols, and quantificational operators (esp. *all* and *some*) to reflect this kind of ontological scenario. The new concern here is with arguments that involve quantificational as well as truthfunctional operations. Firstorder logic is the strongest finitary logic that is both complete and has the Löwenheim–Skolem property. The latter indicates the expressive weakness of firstorder logic insofar as many key mathematical concepts (infinity, continuity, etc.) are therefore not categorically definable in any firstorder language.